Undrained Shear Strength of Unsaturated Soils under Zero or Low Confining Pressures in the Vadose Zone

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In geotechnical engineering practice, many projects such as stability analysis of slopes or trenches and pavement design involve soils in vadose zones at shallow depths, where soils are under unsaturated conditions. For these types of projects, undrained shear strength test results obtained under zero or low confining pressure can be effectively used to analyze the behaviors of unsaturated soils. Experimental procedures to determine the undrained shear strength for different soil suction values, however, is time consuming even under low confining pressure. For this reason, we attempted to develop a semi-empirical model to predict the undrained shear strength of unsaturated soils at a shallow depth assuming zero confining pressure as a function of soil suction. In addition, existing empirical or semi-empirical models are also presented along with the characteristic behaviors of unsaturated soils under axial forces at low or zero confining pressures. Finally, the advantages and disadvantages of each model are discussed based on the comparison between the measured undrained shear strengths under zero or low confining pressures with those predicted using the existing and proposed models.

Abbreviations: SWCC, soil-water characteristic curve; UC, unconfined compression; UCS, unconfined compressive strength; USS, undrained shear strength; VG, van Genuchten.

In conventional geotechnical engineering practice, soil behavior in earthwork projects such as foundation design, slope or trench stability analysis, and retaining wall design are analyzed based on the assumptions that (i) soils are in a saturated condition and (ii) drainage conditions are governed by either a perfectly drained condition or an undrained condition in terms of pore-water pressure. However, there are limitations in adopting the conventional approaches in geotechnical engineering practice because most earthworks predominantly occur in vadose zones where soils exist under unsaturated conditions. When soils get desaturated, the voids that were initially occupied by fluid are replaced with free air. This leads to a change in soil suction, which plays a key role in controlling the mechanical behavior of soils.

For unsaturated coarse-grained soils, the shear strength can be reliably analyzed based on the assumption that both pore air and pore water are under drained conditions. In other words, no change in the soil suction takes place throughout the shearing process until the soils reach failure conditions. On the other hand, there are difficulties in predicting the shear strength of unsaturated fine-grained soils due to the low air and water permeability. In this case, soil suction continuously changes throughout the shearing process, and the difference in soil suction between the initial and failure conditions can be positive, negative, or negligible. These characteristic behaviors encourage the use of a total stress approach instead of an effective stress approach in analyzing the mechanical behavior of unsaturated fine-grained soils.

The shear strength of unsaturated soils can be determined experimentally using the various laboratory testing methods such as consolidated drained, constant water content, unconsolidated undrained, and unconfined compression (UC) tests. Among these tests, the UC test can be the most reasonable choice in estimating the undrained shear strength of unsaturated fine-grained soils, extending the total stress approach. This is...
not only because the UC test is the simplest and most economical, but many projects in geotechnical and pavement engineering involve soils at shallow depths (i.e., stability of natural, expansive, or embankment slopes and pavement design) (American Association of State Highway and Transportation Officials, 1993; Whelham et al., 2007; Li and Zhang, 2015). Oh and Vanapalli (2013) performed model footing tests in Indian Head till for different soil suction values to investigate the bearing capacity of unsaturated fine-grained soils. They showed that half of the unconfined compressive strength (UCS) of unsaturated soils can be used as an undrained shear strength of unsaturated fine-grained soils without introducing significant error in the total stress approach for footings near the soil surface.

Although UC tests are performed under zero confining pressure, it is still challenging to prepare the soil specimens at different soil suction values without causing notable changes to the soil structure. For this reason, we attempted to develop a semi-empirical model to predict the undrained shear strength of unsaturated fine-grained soils in vadose zones under zero or low confining pressure as a function of the soil suction based on the UCS of unsaturated fine-grained soils. In addition, existing empirical or semi-empirical models are also presented along with the characteristic behaviors of unsaturated soils under axial forces under zero or low confining pressure. Finally, the advantages and disadvantages of each model are discussed by comparing the measured undrained shear strengths with those predicted using various existing models.

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## Models

### Behavior of Unsaturated Soils under Zero or Low Confining Pressures

The UC test is the most commonly used laboratory test to determine the undrained shear strength of soils due to its simplicity. For a saturated soil, UC tests are conducted with a relatively fast shearing rate (typically 1% of a specimen height per minute) to achieve undrained conditions in terms of the pore water, and the results are analyzed assuming zero volume change (i.e., the internal friction angle of a saturated soil from an unconsolidated undrained test \( \phi_u = 0 \) analysis). However, the soils encountered in geotechnical projects are typically under unsaturated conditions and the behavior of unsaturated soils during UC tests is different from that of saturated soils. This is mainly because, unlike for saturated soils, volume changes are inevitable in unsaturated soils even under undrained conditions since volume changes in unsaturated soils are associated with not only the volume of water but also the volume of air. This phenomenon, in turn, leads to a change in the soil suction during shearing under undrained conditions. Possible stress paths during the shearing process under zero confining pressure for unsaturated soils are well explained in Fig. 1. For a known initial suction value, the stress state of a soil specimen for a UC test can be represented as the point \( s_i \) under zero confining pressure before shearing. During the shearing process, the soil suction can increase, decrease (stress path \( s_i \rightarrow s_{f-d} \)), or remain constant (stress path \( s_i \rightarrow s_{f-c} \)).

Fredlund et al. (1978) proposed an equation for the shear strength of unsaturated soils based on the extended Mohr–Coulomb failure envelope:

\[
\tau_{fi} = \epsilon' + (\sigma_f - u_a)\tan \phi' + (u_a - u_w) \frac{\tan \phi}{\tan \phi_b}
\]

where \( \tau_{fi} \) is the shear strength of an unsaturated soil, \( \epsilon' \) is effective cohesion, \( \phi' \) is effective internal friction angle, \( (\sigma_f - u_a) \) is net normal stress at failure, \( (u_a - u_w) \) is matric suction at failure, \( u_a \) is pore-air pressure, \( u_w \) is pore-water pressure, and \( \phi_b \) is the internal friction angle due to the contribution of matric suction.

If soil suction does not change throughout the UC test, the undrained shear strength of an unsaturated soil can be approximated as half of the UCS without introducing significant error (Fig. 2). In the present study, half of the UCS of an unsaturated soil, \( q_{u(unsat)}/2 \), is defined as the undrained shear strength (USS) of an unsaturated soil with the notation of \( c_{u(unsat)} \):

\[
q_{u(unsat)}/2 = c_{u(unsat)} \approx \epsilon' + (\sigma_f - u_a)\tan \phi' + (u_a - u_w) \frac{\tan \phi}{\tan \phi_b}
\]

where \( q_{u(unsat)} \) is the UCS of an unsaturated soil and \( c_{u(unsat)} \) is the undrained shear strength of an unsaturated soil.

Characteristic behaviors of unsaturated soils subjected to axial forces under zero or low confining pressure have been studied by several researchers. Some of the previous research available in the literature is presented in this study with emphasis on the stress–strain behavior and the changes in soil suction and volume during the shearing process.

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**Fig. 1.** Stress paths during shearing process under zero confining pressure for unsaturated soils (\( s_i \rightarrow s_{f-d} \): no change in soil suction; \( s_i \rightarrow s_{f-c} \): decrease in soil suction) (modified after Fredlund and Rahardjo, 1993).
Pineda and Colmenares (2005)

Pineda and Colmenares (2005) performed axial load tests on statically compacted kaolinite specimens (plasticity index $I_p = 38\%$; diameter = 50 mm, height = 100 mm) under low confining pressure (net normal stress $s_3 - u_a = 5$ kPa) to study the behaviors of unsaturated fine-grained soils for different soil suction values. A conventional triaxial cell apparatus was modified for the tests such that the pore-water and pore-air pressures in the specimens could be controlled and measured. Suction values in the specimens before shearing ranged between 25 and 400 kPa, which corresponds to a degree of saturation in the range of 75.10 to 90.35%. Changes in the soil suction during shearing were monitored, extending the axis-translation technique (Hilf, 1956). Stress–strain relationships showed that specimens behaved like either normally consolidated soils or overconsolidated soils for the initial soil suction values less than or greater than 100 kPa, respectively. The ratios of soil suction values at failure to the initial values were approximately in the range of 0.9 to 1.6, which decrease as the initial soil suction increases.

Chae et al. (2010)

Chae et al. (2010) conducted UC tests on non-plastic silty soil samples prepared by both dynamic and static compaction (diameter = 50 mm, height = 120 mm). The UC test results only for dynamically compacted soils (optimum, drier than optimum, and wetter than optimum) are presented here. The rate of strain used for the UC tests was 0.1% per minute. Initial soil suction values in the specimens ranged between 0 and 40 kPa (corresponding degree of saturation = 40–80%). The specimen compacted drier than optimum exhibited negligible change in suction at failure (less than about 5 kPa); however, soil suction in the specimens compacted at optimum and wetter than optimum increased up to about 23 kPa at failure. Volumetric strains at failure were approximately between −3 and 2%.

Li and Zhang (2015)

Li and Zhang (2015) investigated the stress–strain behavior of unsaturated soils (compacted Fairbank silt, $I_p = 3.1\%$) under low confining pressures ($s_3 - u_a = 5$ and 40 kPa) at a constant rate of strain (i.e., 0.3 mm/min). Two high-suction tensiometers (Ridley and Burland, 1993) were used to monitor the soil suction during shearing. Initial suction values used in the testing program ranged from 51.1 to 604.9 kPa. Most specimens experienced a continuous decrease in suction, with a maximum change of 17% at failure. All tested specimens failed at axial strains of <2%. Initial specific volumes of the specimens (1.711–1.734 m$^3$/m$^3$) and those at failure (1.704–1.752 m$^3$/m$^3$) indicated negligible volume changes throughout the tests.

These UC test results for unsaturated soils clearly indicate that (i) matric suction can increase, decrease, or remain constant at failure, and (ii) volume change at failure may not be significant.

Estimating Undrained Shear Strength of Unsaturated Soils Based on Unconfined Compressive Strength

Figure 3 shows the USS of unsaturated soils for different suction values and best-fitting curves using hyperbola and power models (Table 1). Most data sets can be well plotted using the hyperbola model (Eq. [3]) with high $R^2$ values. In the case of the data of Vanapalli et al. (2000; Fig. 3b) and Babu et al. (2005; Fig. 3c), although $R^2$ values are relatively high, the hyperbola model does not capture the behaviors of the USS in the low suction range; instead, a power model (three-parameter function; Eq. [4]) provides better fits including the USS at low suction range:

$$C_u(\text{unsat}) = \frac{y_0 + h_1 \psi}{1 + h_2 \psi}$$  \[3\]

$$C_u(\text{unsat}) = y_0 + p_1 \psi^{p_2}$$  \[4\]

where $y_0$ is the $y$ intercept; $h_1$, $h_2$, $p_1$, and $p_2$ are parameters; and $\psi$ is soil suction.
However, the parameters $h_1$ and $h_2$ (Eq. [3]) and $p_1$ and $p_2$ (Eq. [4]) summarized in Table 1 do not show any rules or trends, which are the keys in developing empirical or semi-empirical models. Here, the existing methodologies and approaches to predict the UCS for different suction values are introduced.

Aitchison (1957)

Aitchison conducted UC tests for unsaturated heavy clays and proposed an equation to predict the UCS of unsaturated soils by extending the research of Bjerrum (1954):

$$\Delta \left( \log_{10} C_e \right) / \epsilon = \text{constant}$$ [5]

where $\Delta \left( \log_{10} C_e \right)$ is the change in true cohesion $C_e$, and $\epsilon$ is the void ratio. Equation [5] indicates that true cohesion and water content in any soil can be represented as a linear relationship on a semi-logarithmic scale. Equation [6] shows the pressure vs. void ratio relationship of the consolidation process:

$$\Delta \left( \log_{10} p \right) / \epsilon = \text{constant}$$ [6]

where $\Delta \left( \log_{10} p \right)$ is the change in effective consolidation pressure, $p$. Combining Eq. [5] and [6] yields

$$C_e = kp^M$$ [7]

where $k$ and $M$ are constants.

Extending Eq. [7], the UCS of an unsaturated soil can be predicted using

$$q_u^{(\text{unsat})} = 2\sigma_u^{(\text{unsat})} = 2K(\sigma_u^+)^m = 2K(\psi)^m$$ [8]

where $\sigma_u^+$ is ambient effective stress, $\psi$ is soil suction, and $K$ and $m$ are coefficients.

Aitchison (1957) investigated the coefficients $K$ and $m$ using UC test results for seven heavy clays; however, no trends were observed (see Fig. 4). To further investigate Eq. [8], the six sets of UC test results for unsaturated soils (data in Table 1) available in the literature were analyzed in this study. The $K$ and $m$ coefficients of Aitchison (1957) for each data set are summarized in Table 2. No correlations were observed when $K$ and $m$ values were plotted against $I_p$ (Fig. 5). The coefficient $m$ seems to be decreasing with increasing $K$, but the trend is too weak to develop any correlations (Fig. 6).

Vanapalli and Fredlund (1997)

The shear strength equation for unsaturated soils (i.e., Eq. [1]) can be rewritten as Eq. [9] and then modified as Eq. [10] to estimate the UCS of unsaturated soils by setting $\sigma_3 = 0$ (UC test) and replacing $\tan \phi^b$ with $\phi^b \tan \phi'$ (Vanapalli et al., 1996; Vanapalli and Fredlund, 1997):

$$\frac{\sigma_1 - \sigma_3}{2} = \epsilon' \cos \phi' + \left( \frac{\sigma_1 + \sigma_3}{2} - \sigma_u \right) \sin \phi' + (\sigma_u - u_w) \tan \phi' \cos \phi'$$ [9]
where \( Q \) is the normalized volumetric water content (\( = q / q_s \), where \( q \) is the volumetric water content and \( q_s \) is the volumetric water content under saturated conditions), and \( k \) is a fitting parameter for shear strength (Garven and Vanapalli, 2006; Fig. 7).

Equation [10] was successfully used to predict the UCS of an unsaturated silty soil (Vanapalli et al., 2000) for suction values in the range of 0 to 10^6 kPa. Equation [10] can also be used to estimate \( f_b \) of an unsaturated soil based on UC test results (Vanapalli and Fredlund, 1997).

Babu et al. (2005) examined the UCS of Red and Black Cotton soils and suggested that the UCS of unsaturated soils can be written in the form

\[
\sigma_{u(\text{unsat})} = \frac{c'}{2} \cos \phi' + \left( u_a - n_w \right) \left( \Theta^e \right) \tan \phi' \cos \phi' \quad \text{[10]}
\]

where \( \Theta \) is the normalized volumetric water content (\( = \theta / \theta_r \), where \( \theta \) is the volumetric water content and \( \theta_r \) is the volumetric water content under saturated conditions), and \( \phi' \) is a fitting parameter for shear strength (Garven and Vanapalli, 2006; Fig. 7).

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Babu et al. (2005) examined the UCS of Red and Black Cotton soils and suggested that the UCS of unsaturated soils can be written in the form

\[
\alpha_0 e^\alpha_1 - \alpha_1 = \left( u_a - n_w \right) \left( \Theta^e \right) \alpha_2
\]

where \( \alpha_0, \alpha_1, \) and \( \alpha_2 \) are functions of the strength parameters of soils (i.e., \( c' \) and \( \phi' \)).

To further investigate the validity of Eq. [10], UC test results presented by Cunningham et al. (2003) were analyzed (\( I_p = 18\% \)) in the present study. The soil-water characteristic curve (SWCC) of the soil is shown in Fig. 8. Best-fit analysis was conducted using the van Genuchten (1980) equation (i.e., the VG model):

\[
S_{e} = \left( \frac{\theta - \theta_r}{\theta_i - \theta_r} \right)^m = \left( 1 + \left( \alpha \psi \right)^n \right)^{-m}
\]

where \( S_{e} \) is the effective degree of saturation, \( \theta_i \) is the volumetric water content under saturated conditions, \( \theta_r \) is the residual

Table 1. Models and parameters used for the best-fitting analysis for the unconfined compression test results for unsaturated soils.

<table>
<thead>
<tr>
<th>Reference</th>
<th>( I_p )</th>
<th>( \gamma_0 )</th>
<th>( \beta_1 )</th>
<th>( \beta_2 )</th>
<th>( P_1 )</th>
<th>( P_2 )</th>
<th>( R^2 )</th>
</tr>
</thead>
<tbody>
<tr>
<td>Hyperbola model (Eq. [3])</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Vanapalli et al. (2000)</td>
<td>8</td>
<td>78.8</td>
<td>1370</td>
<td>66,100</td>
<td>0.95</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Vanapalli et al. (2007)</td>
<td>15.5</td>
<td>12.5</td>
<td>87.8</td>
<td>147</td>
<td>0.98</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Ridley (1993)</td>
<td>32</td>
<td>7.32</td>
<td>1210</td>
<td>4004</td>
<td>0.98</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Chen (1984)</td>
<td>38</td>
<td>63.1</td>
<td>1.07 \times 10^9</td>
<td>3.02 \times 10^9</td>
<td>0.99</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Pineda and Colmenares (2005)</td>
<td>38</td>
<td>19.9</td>
<td>272</td>
<td>1105</td>
<td>0.99</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Babu et al. (2005)</td>
<td>60</td>
<td>14.6</td>
<td>75.2</td>
<td>4106</td>
<td>0.95</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Power model (Eq. [4])</td>
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<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Vanapalli et al. (2000)</td>
<td>8</td>
<td>15.6</td>
<td></td>
<td>6.53</td>
<td>0.425</td>
<td>0.98</td>
<td></td>
</tr>
<tr>
<td>Vanapalli et al. (2007)</td>
<td>15.5</td>
<td>12.0</td>
<td></td>
<td>2.68</td>
<td>0.559</td>
<td>0.97</td>
<td></td>
</tr>
<tr>
<td>Ridley (1993)</td>
<td>32</td>
<td>3.81</td>
<td></td>
<td>0.892</td>
<td>0.811</td>
<td>0.98</td>
<td></td>
</tr>
<tr>
<td>Chen (1984)</td>
<td>38</td>
<td>64.0</td>
<td></td>
<td>0.272</td>
<td>1.043</td>
<td>0.99</td>
<td></td>
</tr>
<tr>
<td>Pineda and Colmenares (2005)</td>
<td>38</td>
<td>19.4</td>
<td></td>
<td>0.485</td>
<td>0.839</td>
<td>0.99</td>
<td></td>
</tr>
<tr>
<td>Babu et al. (2005)</td>
<td>60</td>
<td>13.0</td>
<td></td>
<td>1.52</td>
<td>0.388</td>
<td>0.98</td>
<td></td>
</tr>
</tbody>
</table>

Table 2. Plasticity index \( I_p \) and coefficients \( K \) and \( m \) of Aitchison (1957) for various soils based on unconfined compression test results.

<table>
<thead>
<tr>
<th>Reference</th>
<th>( I_p )</th>
<th>( K )</th>
<th>( m )</th>
</tr>
</thead>
<tbody>
<tr>
<td>%</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Vanapalli et al. (2000)</td>
<td>8</td>
<td>16.5</td>
<td>0.41</td>
</tr>
<tr>
<td>Vanapalli et al. (2007)</td>
<td>15.5</td>
<td>25.3</td>
<td>0.3</td>
</tr>
<tr>
<td>Ridley (1993)</td>
<td>32</td>
<td>1.77</td>
<td>0.81</td>
</tr>
<tr>
<td>Chen (1984)</td>
<td>38</td>
<td>0.88</td>
<td>0.63</td>
</tr>
<tr>
<td>Pineda and Colmenares (2005)</td>
<td>38</td>
<td>11.15</td>
<td>0.46</td>
</tr>
<tr>
<td>Babu et al. (2005)</td>
<td>60</td>
<td>11.8</td>
<td>0.26</td>
</tr>
</tbody>
</table>
volumetric water content, and \( \alpha, n, \) and \( m = 1 - 1/n \) are fitting parameters. The parameters are shown in Fig. 8.

Figure 9 shows the measured and predicted (Eq. [10]) UCS for different suction values using the data of Cunningham et al. (2003). According to Garven and Vanapalli (2006), \( k \) ranges from 1 to 3 and \( k = 2.23 \) can be used for \( I_p = 18\% \). The value \( k = 1 \) provided better estimates when compared with the measured UCS than those obtained using \( k = 2.23 \) or 3. This indicates that the predicted UCS using Eq. [10] is sensitive to the fitting parameter \( k \).

Chae et al. (2010) suggested that there is a unique relationship between the UCS and suction stress at failure. Stress components resulting from bulk water and meniscus water are termed bulk \( (p_b) \) and meniscus \( (p_m) \) stresses, respectively. The sum of these two stresses is denoted as the suction stress \( (p_s) \) (Karube and Kato, 1996). The suction stress for different suction values can be estimated by adopting the VG model (Eq. [12]; Lu and Griffiths, 2004) and using

\[
p_s = p_b + p_m = \left[ \frac{1}{1 + (\alpha \psi)^n} \right]^m \psi
\]

Suction stress is located on the axis of net normal stress (i.e., \( \sigma - u_a \)) as an intercept (Karube et al., 1996; Lu and Likos, 2006). Therefore, if it is assumed that suction stress at failure acts as a confining pressure during a UC test, an equation can be derived based on Mohr’s circle at failure (Fig. 10):

\[
g_{\text{u(unsat)}} = \frac{4 \sin \phi'}{1 - \sin \phi'} P_{ci}
\]

where \( P_{ci} \) is the suction stress at failure.

Fig. 8. Soil-water characteristic curve for the soil used by Cunningham et al. (2003).
The internal friction angle in Eq. [14] is based on the net normal stress. There is a limitation to the use of Eq. [14] because predicting the suction stress at failure is still challenging.

Zhou et al. (2016)

A similar approach to that of Chae et al. (2010) was also used by Zhou et al. (2016) to predict the UCS of unsaturated soils. The unsaturated soil effective stress theory of Bishop (1959) is

\[ \sigma' = (\sigma - u_s) + \chi (u_s - u_w) \]  

[15]

Because \( \sigma \) is zero during UC tests, the effective horizontal and vertical stresses at failure conditions become \( \sigma'_h \) and \( \sigma'_v \), respectively:

\[ \sigma'_h = \chi \psi_f; \quad \sigma'_v = \chi \psi_f + q_u \]  

[16]

where \( \psi_f \) is the soil suction at failure.

The effective stress parameter, \( \chi \), in Eq. [15] can be evaluated using the approach of Khalili and Khabbaz (1998):

\[
\chi = \begin{cases} 
\left( \frac{u_s - u_w}{u_s - u_w} \right)_b & \text{if } (u_s - u_w) > (u_s - u_w)_b \\
1 & \text{if } (u_s - u_w) \leq (u_s - u_w)_b 
\end{cases}
\]  

[17]

where \( (u_s - u_w)_b \) is the air-entry value of a soil.

By replacing \( p_{sf} \) and \( p_{sf} + q_u \) in Fig. 10 with \( \chi \psi_f \) and \( \chi \psi_f + q_u \), respectively, the UCS for different matric suction values can be obtained using

\[ q_{u(unsat)} = -\chi (\delta_1 \psi_f + \delta_2) \sin \phi' - c' \cos \phi' \]  

\[ 0.5(\sin \phi' - 1) \]  

[18]

where \( \delta_1 \) and \( \delta_2 \) are linear relationship fitting parameters and \( \psi_f \) is the initial soil suction.

The disadvantage of the approaches proposed by Chae et al. (2010) and Zhou et al. (2016) are that they require not only shear strength parameters for saturated conditions (i.e., \( c' \) and \( \phi' \)) but also the suction value at failure.

Proposed Model

According to Han and Vanapalli (2016), various types of equations are available to predict the stiffness and shear strength properties of unsaturated soils, and one of them can be written in the form (Vanapalli et al., 1996; Gupta et al., 2007; Sawangsuriya et al., 2009; Oh et al., 2009)

\[ \Omega = \Omega_{sat} + \Gamma \psi (S_r)^\xi \]  

[20]

where \( \Omega \) is the stiffness and shear strength properties of an unsaturated soil, \( \Omega_{sat} \) is the stiffness and shear strength properties of unsaturated soils at saturation, \( \Gamma \) is a suction-related variable, \( \psi \) is soil suction, \( S_r \) is the degree of saturation, and \( \xi \) is an exponent.

The exponent \( \xi \) in Eq. [20] is empirical in nature, and regression analysis is necessary to determine the value. Some researchers have suggested using the effective degree of saturation instead of the degree of saturation, as shown in Eq. [12], to avoid the regression analysis procedure. The degree of saturation can replace the effective degree of saturation in the case where a soil sample experiences relatively small volume change during tests (Romero and Vaunat, 2000; Tarantino and Tombolato, 2005; Alonso et al., 2010; Lu et al., 2010):

\[ S_r = \frac{S_r - S_{r0}}{100 - S_{r0}} \]  

[21]
where \( S_e \) is the effective degree of saturation, \( S_r \) is the degree of saturation, and \( S_{r0} \) is degree of residual saturation.

Adopting the form of Eq. [20] and extending the concept in Eq. [2] and Fig. 2, the USS of an unsaturated soil can be estimated using either

\[
\epsilon_{u(unsat)} = \frac{q_{u(unsat)}}{2} = \epsilon_{u(sat)} + \Gamma_{UC} \psi \left( S_r \right)^p \tag{22}
\]

or

\[
\epsilon_{u(unsat)} = \frac{q_{u(unsat)}}{2} = \epsilon_{u(sat)} + \Gamma_{UC} \psi \left( S_e \right) \tag{23}
\]

where \( \Gamma_{UC} \) is a suction-related variable based on UC test results, and \( p \) is an exponent.

If the variable \( \Gamma_{UC} \) in Eq. [22] and [23] can be written in a form \( \epsilon_{u(sat)/\mu} \left( P_a/101.3 \right) \), where \( \mu \) is a function of the plasticity index of soil properties, \( P_a \) is the atmospheric pressure (in kPa), and \( (P_a/101.3) \) is used to maintain the consistency in units, Eq. [22] and [23] can be rewritten, respectively, as

\[
\epsilon_{u(unsat)} = \epsilon_{u(sat)} + \epsilon_{u(sat)} \left( 1 - \frac{P_a}{101.3} \right) \left( S_r \right) \tag{24}
\]

\[
\epsilon_{u(unsat)} = \epsilon_{u(sat)} + \epsilon_{u(sat)} \left( 1 + \frac{\psi}{\mu} \left( \frac{P_a}{101.3} \right) \left( S_e \right) \right) \tag{25}
\]

Although Eq. [25] requires one fewer parameter (i.e., \( p \)), more suction measurements are necessary to reliably estimate the residual degree of saturation based on the entire range of the SWCC. Hence, in the present study, Eq. [24] was used to develop a model to estimate the variation in the USS with respect to soil suction.

There is a fundamental flaw in Eq. [24] because, as explained above, unsaturated soils undergo volume changes due to the compressibility of an air–water mixture even under undrained conditions while the volume change during UC tests for saturated soils is neglected. This critical difference in the behaviors of saturated and unsaturated soils during UC tests indicates that the USS under saturated conditions, \( \epsilon_{u(sat)} \), should be replaced with a certain reference value, \( \epsilon_{u(ref)} \):

\[
\epsilon_{u(unsat)} = \epsilon_{u(ref)} + \epsilon_{u(ref)} \left( 1 + \frac{\psi}{\mu} \left( \frac{P_a}{101.3} \right) \left( S_r \right) \right) \tag{26}
\]

Toll (1990) proposed that the shear strength of unsaturated soils can be estimated based on the critical states using

\[
q = M_a \left( p - u_a \right) + M_w \left( u_a - u_w \right) \tag{27}
\]

where \( p - u_a \) is the net mean stress, \( M_a \) is the total stress ratio, and \( M_w \) is the suction ratio (both \( M_a \) and \( M_w \) are functions of the degree of saturation).

Wheeler (1991) stated that Eq. [27] is useful to analyze existing experimental data but not practical since there is no way of predicting the variation of \( S_r \) during tests. He then suggested that the shear strength of an unsaturated soil can be estimated by uncoupling two variables, \( p - u_a \) and \( u_a - u_w \):

\[
q = M \left( p - u_a \right) + f \left( u_a - u_w \right) \tag{28}
\]

where \( M \) is the critical state stress ratio for saturated soils and \( f (u_a - u_w) \) is a function of suction.

Pineda and Colmenares (2005) analyzed the UCS results for compacted kaolin samples and showed that the UCS can be predicted by extending the approach of Wheeler (1991) (i.e., Eq. [28]):

\[
q_{u(unsat)} = M \left( p - u_a \right) + f \left( u_a - u_w \right)
+ \left[ 0.4208 \left( u_a - u_w \right) - 0.0003 \left( u_a - u_w \right)^2 + 23.35 \right] \tag{29}
\]

The use of \( M \) (i.e., 0.81 for saturated conditions) and the \( y \) intercept (i.e., 23.35) indicate that, although the behaviors of saturated and unsaturated soils during UC tests are different, unconfined compression test results for saturated conditions can be used as a reference strength to estimate the shear strength of an unsaturated soil for different suction values.

\( \Box \) Validation of the Proposed Model

To check the validity of the proposed model, six sets of UC test results for compacted fine-grained soils (Table 1) were analyzed. Figure 11 shows the SWCCs of the soils. Best-fit analyses
were conducted using the VG model (Eq. [12]), and the fitting parameters for each SWCC are summarized in Table 3.

**Determination of the Fitting Parameter \( \nu \)**

The fitting parameter \( \nu \) controls the variation in the USS of unsaturated soils in a nonlinear pattern. To investigate the fitting parameter, \( \nu \) values for two sets of UC test results were used—one for the lowest \( I_p = 8\% \) (Vanapalli et al., 2000) and another one for the highest \( I_p = 60\% \) (Babu et al., 2005) from Table 1. Figure 12a shows the measured and predicted undrained shear strengths for different suction values using the UC test results of Vanapalli et al. (2000). Combination of \( \nu = 2 \) and \( \mu = 10 \) provided good estimates, and then \( \mu \) increased with increasing \( I_p \) value. This behavior is shown in

\[
\begin{align*}
\mu &= 10 \quad \text{for } 8 \leq I_p(\%) \leq 15.5 \\
\mu &= 1.77 \exp \left[ 0.0937 \left( I_p \right) \right] \quad \text{for } 15.5 \leq I_p(\%) \leq 60
\end{align*}
\]

The point obtained using the UC test results of Cunningham et al. (2003) is explained below.

**Initial Soil Suction vs. Soil Suction at Failure**

As pointed out by Chae et al. (2010) and Zhou et al. (2016), the UCS of unsaturated soils can be more reliably predicted when soil suction at failure is used. Figure 19 shows five different plots of soil suction vs. USS behaviors considering both initial and failure conditions as detailed below using the UC test results of Cunningham et al. (2003): the measured USS vs. \( \psi_i \), the measured USS vs. \( \psi_f \), the predicted USS using \( S_r \) and \( \psi_i \), the predicted USS using \( S_r \) and \( \psi_f \), and the predicted USS using \( S_r \) and \( \psi_f \), where subscript \( i \) = initial and \( f \) = failure.

The results show that the USS values plotted against soil suction at failure are in good agreement when compared with the USS

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**Table 3. Van Genuchten model parameters for the various soils analyzed in this study.**

<table>
<thead>
<tr>
<th>Reference</th>
<th>( \alpha )</th>
<th>( n )</th>
<th>( m = (1 - 1/n) )</th>
</tr>
</thead>
<tbody>
<tr>
<td>Vanapalli et al. (2000)</td>
<td>( 1.0023 \times 10^{-2} )</td>
<td>1.300</td>
<td>0.23077</td>
</tr>
<tr>
<td>Vanapalli et al. (2007)</td>
<td>( 7.6576 \times 10^{-2} )</td>
<td>1.511</td>
<td>0.33819</td>
</tr>
<tr>
<td>Ridley (1993)</td>
<td>( 6.2100 \times 10^{-4} )</td>
<td>8.911</td>
<td>0.88778</td>
</tr>
<tr>
<td>Chen (1984)</td>
<td>( 5.7531 \times 10^{-3} )</td>
<td>1.522</td>
<td>0.34297</td>
</tr>
<tr>
<td>Pineda and Colmenares (2005)</td>
<td>( 5.5388 \times 10^{-2} )</td>
<td>1.251</td>
<td>0.20045</td>
</tr>
<tr>
<td>Babu et al. (2005)</td>
<td>( 4.4403 \times 10^{-3} )</td>
<td>1.220</td>
<td>0.18033</td>
</tr>
</tbody>
</table>
predicted using $S_{ri}$ and $\psi_i$ or $S_{rf}$ and $\psi_f$. This indicates that the proposed model that uses $S_{ri} + \psi_i$ can predict the variation of the USS with respect to soil suction without knowing $S_{rf}$ and $\psi_f$. This is attributed to the reason that the increase (or decrease) in soil suction at failure leading to a decrease (or increase) in the degree of saturation, which minimizes the difference in USS predicted using $S_{ri}$ and $\psi_i$ or $S_{rf}$ and $\psi_f$.

**Limitations of the Proposed Model**

The model proposed in the present study was validated using limited sets of UC test results for unsaturated fine-grained soils whose $I_p$ values are between 8 and 60% for drying paths. Hence, more UC test results need to be analyzed to increase the reliability of the proposed model. The measured and predicted USS values for various suction values using the data of Ridley (1993; Fig. 15) showed that the discrepancy increases as soil suction approaches the residual suction value. Similar behavior was observed for the

---

Fig. 13. Measured and predicted (Eq. [24]) undrained shear strength (USS) with different sets of $\nu$ and $\mu$ (data from Babu et al., 2005).

Fig. 14. Measured and predicted (Eq. [24]) undrained shear strength (USS) (data from Vanapalli et al., 2007).

Fig. 15. Measured and predicted (Eq. [24]) undrained shear strength (USS) (data from Ridley, 1993).

Fig. 16. Measured and predicted (Eq. [24]) undrained shear strength (USS) (data from Chen, 1984).

Fig. 17. Measured and predicted (Eq. [24]) undrained shear strength (USS) (data from Pineda and Colmenares, 2005).
data of Aitchison (1957; Fig. 20). This is because, theoretically, the net contribution of soil suction toward the shear strength of soils starts decreasing as the soil suction approaches the residual suction value (Vanapalli et al., 1996). In other words, the proposed model may not provide good estimates of the USS for suction values greater than the residual suction value.

It also should be noted that the sample volume change during wetting–drying cycles or the shearing process was not taken into account in developing the proposed model (i.e., Eq. [24]). As mentioned above, volume changes are inevitable in unsaturated soils even under undrained conditions (i.e., in spite of a constant water content condition). Measurement of the volume change during wetting–drying cycles or the shearing process is one of the major challenges in unsaturated soil testing, which can be achieved with double cell or on-sample strain transducers. However, as described above, the proposed model provides reasonable estimation of the USS of unsaturated cohesive soils using information on only the initial soil suction and degree of saturation.

### Table 4. Plasticity index $I_p$ and the fitting parameter $\mu$ of the proposed model using an exponent $\nu = 2$ for various soils based on unconfined compression test results.

<table>
<thead>
<tr>
<th>Reference</th>
<th>$I_p$</th>
<th>$\mu$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Vanapalli et al. (2000)</td>
<td>8</td>
<td>10</td>
</tr>
<tr>
<td>Vanapalli et al. (2007)</td>
<td>15.5</td>
<td>10</td>
</tr>
<tr>
<td>Ridley (1993)</td>
<td>32</td>
<td>35</td>
</tr>
<tr>
<td>Chen (1984)</td>
<td>38</td>
<td>60</td>
</tr>
<tr>
<td>Pineda and Colmenares (2005)</td>
<td>38</td>
<td>65</td>
</tr>
<tr>
<td>Babu et al. (2005)</td>
<td>60</td>
<td>490</td>
</tr>
</tbody>
</table>

### Fig. 18. Relationship between the plasticity index $I_p$ and the fitting parameter $\mu$.

### Fig. 19. Measured and predicted (Eq. [24]) undrained shear strength (USS) considering both initial ($\psi_i$ and degree of saturation $S_{ri}$) and failure ($\psi_f$ and $S_{rf}$) conditions (data from Cunningham et al., 2003).

### Fig. 20. (a) Soil-water characteristic curve and (b) measured and predicted (Eq. [24]) undrained shear strength (USS) (data from Aitchison, 1957).
Summary and Conclusions

Unconfined compressive strengths are widely used in geotechnical practice to estimate the USS of soils. Under field conditions, environmental factors such as water infiltration or evaporation through the surface of a soil can cause variations in soil suction, which in turn leads to changes in the USS. Hence, to numerically model soil behaviors under wetting and drying conditions, it is necessary to use a model that can predict the variation in the USS with respect to soil suction. In the present study, four different empirical or semi-empirical models available in the literature were revisited and their disadvantages discussed. To overcome the disadvantages of the existing models, a new model is proposed to estimate the USS of unsaturated soils for different suction values under zero or low confining pressures following drying paths. The findings from this study can be summarized as follows:

1. The suction value in a compacted soil can increase, decrease, or remain constant during the shearing process under axial forces in an unconfined condition, which makes it more challenging to develop a model to predict the variation of USS with respect to soil suction.

2. The model proposed by Aitchison (1957) has difficulties in correlating the coefficients $K$ and $m$ to soil properties such as the plasticity index. The predicted USS using the equation proposed by Vanapalli and Fredlund (1997) is too sensitive to the fitting parameter $\kappa$. Lastly, the methodologies proposed by Chae et al. (2010) and Zhou et al. (2016) require the soil suction at failure, which is impossible to predict.

3. The proposed model requires the USS under saturated conditions and the soil-water characteristic curve along with two fitting parameters, $\nu$ and $\mu$. However, since $\nu = 2$ provides good comparisons when compared with the measured USS for a wide range of $I_p$, only one fitting parameter (i.e., $\mu$) is required.

4. The predicted USS from the proposed model showed good agreement with the measured USS using the degree of saturation and soil suction under both initial and failure conditions. In other words, the proposed model does not require the degree of saturation and soil suction at failure. The reason for the good agreement between the predicted USS under both initial and failure conditions is that the change in the soil suction at failure is compensated by the change in the degree of saturation.

5. The USS predicted using the proposed model decreases as soil suction approaches the residual suction value even though the USS gradually increases or remains constant for suction values greater than the residual suction value. Hence, the proposed model may not be applicable to the suction values greater than the residual suction value.

References


Alonso, E.E., J.M. Pereira, J. Vaunat, and S. Olivella. 2010. A microstruc-


